Why did Cournot's principle disappear?

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1. Geopolitical explanation: The British & Americans never liked it in the first place.

2. Whiggish explanation: They were right. The contrast between subjectivism and stochasticism is not so easily finessed.

3. Counterfactual explanation: Opportunities were missed.

1. The British & Americans never liked it in the first place.
   - Norbert Wiener, philosopher and analyst.
   - Joe Doob, statistician and probabilist.

2. Alternatives were more interesting and convincing.
   - Bruno De Finetti's subjectivism.
   - Jerzy Neyman's stochasticism.

3. Opportunities were missed.
   - Jean Ville could not find his own voice.
   - Andrei Kolmogorov's philosophy was ignored (except by Prokhorov).
   - Even Karl Popper could not see what was essential.

Background: History of Cournot's Principle

1. Invention of the principle: Bernoulli to Lévy & Fréchet

2. Heyday of the principle: 1930–1960

3. Disappearance of the principle: 1960

Bernoulli related mathematical probability to moral certainty is his celebrated *Ars Conjectandi* (1713).

Jakob Bernoulli
1654–1705

"Something is *morally certain* if its probability is so close to certainty that the shortfall is imperceptible."

"Something is *morally impossible* if its probability is no more than the amount by which moral certainty falls short of complete certainty."

How Bernoulli connected probability with the world:

"*Because it is only rarely possible to obtain full certainty, necessity and custom demand that what is merely morally certain be taken as certain.* It would therefore be useful if fixed limits were set for moral certainty by the authority of the magistracy—if it were determined, that is to say, whether 99/100 certainty is sufficient or 999/1000 is required..."

In other words, an event with very small probability will not happen.

Most of Bernoulli's successors, especially d'Alembert and Buffon, discussed moral certainty.

But as d'Alembert's concept of mixed mathematics gave way to a Kantian picture in which the scientist must relate mathematics to the world, the discussion changed.

Cournot rose to the occasion by suggesting that the principle of impossibility is the only way of connecting the mathematical probability to the world.

Antoine Cournot
1801–1877

Maurice Fréchet, 1878–1973, proposed the name *Cournot's principle*.

Cournot discussed both *moral impossibility* (very small probability) and *physical impossibility* (infinitely small probability).

A physically impossible event is one whose probability is infinitely small. This remark alone gives substance—an objective and phenomenological value—to the mathematical theory of probability.

This remark occurs in Cournot's 1843 book. I have not found it repeated, by Cournot or anyone else, in the 19th century.

The remark concerns only phyically impossibility (zero probability), not moral impossibility (small probability).

But elsewhere Cournot repeatedly explains that small probabilities have practical implications (law of large numbers, roof tile falling on passerby).

At the beginning of the 20th century, there was consensus:

- The statistician (like Bernoulli's magistrate) must fix a level of probability to interpret as moral certainty. (Karl Pearson, George Bohlman, Ladislaus von Borkeiwicz, echoed by Markov)

- Zero probability means impossibility. (Anders Wiman, Felix Bernstein)

But also disagreement:

- The French continued to explain that an event of small probability will not happen.

- But the English (Venn) and the Germans (Czuber) were more likely to talk about this as a mistake. Czuber's misgivings were echoed in the 1920s by Meinong and Slutsky.

Aleksandr Chuprov
1874–1926

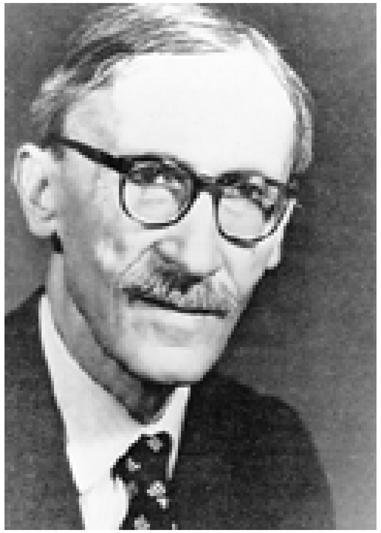
Only Chuprov came close to repeating Cournot's claim that the the principle of moral certainty is the meaning of probability.

Petersburg Polytechnical Institute

In his *Essays on the Theory of Statistics* (in Russian 1909 and 1910), Chuprov called the principle that an event of small probability will not happen Cournot's lemma, because we use it to get from Bernoulli's theorem to the law of large numbers.

It was, he said, the basic principle of the logic of probable.

Paul Lévy was the first to make the point absolutely clear: *Cournot's principle is the only connection between probability and the empirical world.*



Paul Lévy
1886–1971

He first said it clearly in his 1919 course.

In his 1925 book, he explained that probability is based on two principles:

- The principle of equally likely events, which is the foundation for mathematics.
- The principle of the very unlikely event, which is the basis of applications.

Lévy's fellow travellers

- Castelnuovo, *Calcolo delle probabilità*, 1919.
- Hadamard, *Les principes du calcul des probabilités*, 1922.
- Fréchet & Halbwachs, *Le calcul des probabilités à la portée de tous*, 1924
- Kolmogorov, *Grundbegriffe der Wahrscheinlichkeitsrechnung* 1933.
- Borel, *Les probabilités et la vie* 1940, *Probabilité et certitude* 1950.

Jacques Hadamard
1865-1963

The last universal mathematician.

In a 1922 lecture that probably influenced Lévy's 1925 book, Hadamard explained that probability theory is based on two principles:

- The principle of equally likely cases. This is the basis of the mathematics.
- The principle of the negligible event. This connects the mathematics with the real world.

In 1951, Fréchet explained the distinction between the weak and strong forms of Cournot's principle.

- The weak form says an event of small probability seldom happens. It was advanced by Chuprov, Castelnuovo, Fréchet and Halbwachs, Cramér, and Anderson.

- The strong form says an event of small probability will not happen. It was advanced by Cournot, Hadamard, Lévy, Kolmogorov, Borel, and later Richter and Fortet.

But Fréchet and Lévy agreed that Cournot's principle leads to an objective concept of probability: Probability is a physical property just like length and weight.

Andrei Kolmogorov
1903–1987
The Soviet Euler

In his *Grundbegriffe* (1933), Kolmogorov gave two principles for connecting probability with the empirical world:

Principle A: Over many trials, the frequency with which E happens will approximate $\mathrm{P}(E)$.

Principle B: On a single trial, if $\mathrm{P}(E)$ very small, we can be practically certain E will not happen.

According to the weak law of large numbers, B implies A.

Richard von Mises
1883-1953

Von Mises invented the theory of collectives

Kolmogorov acknowledged the influence of Richard von Mises's frequentism. Von Mises said a sequence of trials is random if we not know how to select a subsequence of trials that will be different.

Kolmogorov connected this with Cournot's principle. If an event does not usually happen (because it has small probability), and there is nothing that marks next trial as different, then we can assume the event will not happen on the next trial.

Émile Borel

1871–1956

Inventor of measure theory

Minister of the French navy in 1925

By 1910, Borel was already the uncontested leader of classical French probability. But only in the 1940s was he as clear as Lévy about Cournot's principle being the only link between probability and the world.

Borel's way of saying it: The principle that an event with very small probability will not happen is *the only law of chance*.

- Impossibility on the human scale: $p < 10^{-6}$.
- Impossibility on the terrestrial scale: $p < 10^{-15}$.
- Impossibility on the cosmic scale: $p < 10^{-50}$.

The heyday of Cournot's principle: late 40s, early 50s

- Harald Cramér's *Mathematical Methods in Statistics* followed Kolmogorov's philosophy as well as his mathematics.

- Borel proclaimed his "only law of chance" in the late 40s. Fortet brought it into print in Le Lionnais's *Grands Courants* in 1948. Borel's *Probabilités et certitudes* appeared in 1950.

- At the Congrès international de philosophie des sciences in Paris in 1949, the principle was debated by Anderson, de Finetti, and Neyman and named by Fréchet.

- Continental mathematicians learned the name Cournotsche Prinzip from Hans Richter's 1956 textbook.

The disappearance: late 1950s and 1960s

- English-language enunciations of the principle sound lonely: Marshall 1957 (personal idea), Freudenthal 1960 (what the philosophers say).

- De Finetti repeatedly mocks the principle.

- Martin-Löf learns it from Borel, not Kolmogorov; marvels that it is so neglected.

- Neyman expresses his incomprehension of Anderson and Fisher.

- By the 1970s, only Prokhorov carries Kolmogorov's flame, expressing the principle paradoxically in the Soviet Encyclopedia: only probabilities close to zero or one are meaningful.

By the 1960s, probability is pure. Principles of application belong to the applications.

- In his 1940 Dartmouth debate with von Mises, Doob dismissed philosophy. The application of probability, he said, should be left to the judgement of the statistician.

- Neyman saw significance testing as a principle of statistics, not as part of the meaning of probability

- In the 1970s economists invent an "efficient market hypothesis", unaware that the probabilists had it earlier.

Why did Cournot's principle disappear?

Geopolitical explanation: The British & Americans never liked Cournot's principle in the first place.

- Norbert Wiener, philosopher and analyst.
- Joe Doob, statistician and probabilist.

Norbert Wiener was both philosopher and mathematician.

Studied with Russell. Greatly admired Fréchet. Talked probability with Lévy.

But the philosophy of probability seems not to have interested him.

As Doob said, Wiener was an analyst, not a probabilist. Even more so than Fréchet.

The champion of measure theory

Joseph Doob, 1910–2004, receiving the National Medal of Science from President Carter in 1979.

Picking up where Kolmogorov left off, and systematizing Wiener, Doob showed how continuous random processes (e.g., Brownian motion) can be put in the measure-theoretic framework.

Doob's problem

The philosophical foundation for probability espoused by the French and by Kolmogorov (Bernoulli's theorem + Cournot's principle) breaks down for stochastic processes.

Bernoulli's theorem does not apply because we are not repeating the same random experiment over and over.

Doob could have solved the problem by making Cournot's principle more central than frequentism.

Instead he fled from philosophy.

Why did Cournot's principle disappear?

Whiggish explanation: Cournot's principle is wrong? As the British always thought, probability is about belief and frequency.

- Bruno de Finetti's subjectivism.
- Jerzy Neyman's stochasticism.

The rise of de Finetti's subjectivism has always been a mystery to me.

Why did the French subjectivism (Borel, Lévy, Fréchet) seem so irrelevant?

As de Finetti explained to Fréchet in 1955, he accepted the version of Cournot's principle that says we should act as if an event of very small probability will not happen, but this is only a special case of a rule of action that also applies to middling probabilities.

For probability, the meaning of probability lie in decision, not in testing.

Neyman's solution

After Doob, those who preferred an objective interpretation of probability were less enamored with "probability=frequency".

Often they instead located the meaning of probabilities in their role in generating outcomes.

As Jerzy Neyman explained in a famous article in 1960,

- Laws are needed to produce deterministic phenomena.

- Probabilities are needed to produce indeterministic phenomena.

Indeterministic phenomena exist. Therefore objective probabilities exist.

The contrast between Neyman and Fisher.

Ronald Fisher
1890–1962

In spite of his insistence on frequentism, Fisher still saw the "fiducial" aspect of probability.

For frequency to be probability, you need the absence of selection rules (von Mises) or relevant subsets (Fisher). Neyman did not buy this.

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Opportunities missed.

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Jean Ville,
1910–1988, on
entering the *École
Normale Supérieure*.

In 1939, Ville showed that Cournot's principle can be restated as a principle of market efficiency:

> If you never bet more than you have, you will not get infinitely rich.

Ville's Theorem

Consider binary $Y_1, Y_2, \ldots$ with joint probability distribution P.

Binary Probability Protocol

$\mathcal{K}_0 := 1$.
FOR $n = 1, 2, \ldots$:
  Skeptic announces $s_n \in \mathbb{R}$.
  Reality announces $y_n \in \{0, 1\}$.
  $\mathcal{K}_n := \mathcal{K}_{n-1} + s_n(y_n - \mathrm{P}\{Y_n = 1 | Y_1 = y_1, \ldots, Y_{n-1} = y_{n-1}\})$.

Restriction on Skeptic: Skeptic must choose the s_n so that $\mathcal{K}_n \geq 0$ for all n no matter how Reality moves.

Ville showed that Skeptic's getting infinitely rich in this protocol is equivalent to an event of zero probability happening, in the following sense:

1. When Skeptic follows a measurable strategy (a rule that gives s_n as a function of $y_1, \ldots, y_{n-1}$),

$$\mathsf{P}\{\lim_{n\to\infty} \mathcal{K}_n = \infty\} = 0. \tag{1}$$

2. If A is a measurable subset of $\{0,1\}^\infty$ with $\mathsf{P}(A) = 0$, then Skeptic has a measurable strategy that guarantees

$$\lim_{n\to\infty} \mathcal{K}_n = \infty$$

   whenever $(y_1, y_2, \ldots) \in A$.

But this positive result got little attention.

For Ville's adviser Fréchet, and hence for Ville, Ville's book was interesting only because of its counterexample to von Mises.

Von Mises considered a sequence $y_1, y_2, \ldots$ of 0s and 1s random if no subsequence with a different frequency of 1s can be picked out by a gambler to whom the ys are presented sequentially. This would keep the gambler from getting rich by deciding when to bet.

Ville showed that von Mises's condition is insufficient. It does not rule out the gambler's getting rich by varying the direction and amount to bet.

Joseph Stalin
1879–1953

Connecting probability theory with the real world (statistics) was dangerous under Stalin.

So Kolmogorov stated his philosophy seldom and tersely. Western readers often concluded that he had no philosophy.

Probability is measure, and there is nothing more to say.

Most of the Viennese philosophers who escaped to the US (Carnap, Reichenbach) never digested Cournot's principle.

The exception was Karl Popper, for whom the meaning of all science lay in testing. Yet Popper failed to articulate Cournot's principle clearly, perhaps out of vanity:

- In the English version of *The Logic of Discovery* he spent his time challenging Kolmogorov's axioms.

- Most of his later work was devoted to trying to make "propensity" a novel idea.

- In his 1983 book, he tried to make something out of Dobb's theorem on the impossibility of a gambling strategy.

www.probabilityandfinance.com

- Excerpts from our 2001 book.
- Reviews and responses.
- Working papers.

See especially. . .

- Working Paper # 4. The origins and legacy of Kolmogorov's *Grundbegriffe*.
- Working Paper #15. From Cournot's principle to market efficiency.